Expected Value and Standard Dev.

- Expected Value of a random variable is the mean of its probability distribution
- If $P(X=x_1)=p_1$, $P(X=x_2)=p_2$, …n $P(X=x_n)=p_n$
- $E(X) = x_1*p_1 + x_2*p_2 + \ldots + x_n*p_n$

Properties of E(X)

- $E(X+c) = E(X) + c$ *(for a constant c)*
- $E(aX) = aE(X)$ *(for a constant a)*
- $E(X+Y) = E(X) + E(Y)$ *(for r.v.s X and Y)*

ex) $E(X) = 40$

$E(5X+4) = E(5X)+4 = 5E(X)+4 = 5(40)+4 = 204$

Standard Deviation

- Recall: Variance = Standard Deviation Squared
- So $SD(X) = \sqrt{}Var(X)$
- Let $E(X) = \mu$
- **$Var(X) = E((X-\mu)^2)$**
- Alternate form: $Var(X) = E(X^2) – \mu^2$
- Because $E((X-\mu)^2) = E(X^2–2X\mu+\mu^2)$

$$= E(X^2)–2\mu E(X)+\mu^2$$
$$= E(X^2) –2\mu^2+u^2$$
$$= E(X^2) – \mu^2$$

Properties of Var(X) and SD(X)

- $Var(X+c) = Var(X)$, $SD(X+c) = SD(X)$
- $Var(aX) = a^2 Var(X)$, $SD(aX) = aSD(X)$
- $Var(X+Y) = Var(X)+Var(Y)$
- $SD(X+Y) = \sqrt{}Var(X+Y) = \sqrt{}(Var(X)+Var(Y))$
  $= \sqrt{}(SD(X)^2+SD(Y)^2)$
- **$SD(X+Y) \neq SD(X)+SD(Y)$**
- $SD(X-Y) = SD(X+(-Y))=\sqrt{}(Var(X)+Var(-Y))$
  $=\sqrt{}(Var(X)+Var(Y)) = SD(X+Y)$

Calculating E(X) and Std. Dev

- Given this Probability Distribution, calculate E(X) and SD(X)

x	**P(X=x)**
19	0.4
5	0.3
27	0.2
39	0.1

Calculating (example continued)

- Method 1: Use formula $\sqrt{\mathbf{E((X-\mu)^2)}}$

x	$P(X=x)$	$x-\mu$	$(x-\mu)^2$
19	0.4	0.6	0.36
5	0.3	-13.4	179.56
27	0.2	8.6	73.96
39	0.1	20.6	424.36
	$E(X) =$ 18.4 (i.e. μ)		$E((x-\mu)^2) = 111.24$ $\sigma = \sqrt{111.24} =$ **10.5470**

Calculating (example continued)

- Method 2: Use formula $\sqrt{(\mathbf{E(X^2)–\mu^2})}$

x	P(X=x)	x^2	
19	0.4	361	
5	0.3	25	
27	0.2	729	
39	0.1	1521	
	E(X) = 18.4	$E(X^2)$ = 449.8	$E(X^2)-\mu^2$ = 449.8-338.56 = 110.68 $\sigma = \sqrt{110.68}$ **10.5205**

- Rounding error accounts for the difference